NASA TECHNICAL NOTE

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PRELIMINARY ANALYSIS OF A TITANIUM ALLOY HONEYCOMB SOLAR ABSORBER HAVING BLACKENED WALLS

by William J. Bifano

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ERRATA

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August 1968

In calculating the apparent hemispherical emittance of a honeycomb-solar absorber, it was necessary to solve nonlinear integro-differential equations (eq. (9)) which describe the energy transport within the honeycomb. For TN D-4727, numerical solutions were obtained using a digital computer and a specially devised iterative technique which was formulated in reference 5. Subsequent investigation indicated that this technique was inadequate for length-to-diameter ratios (L/d) of 3 or more. Hence, the results presented in TN D-4727 are incorrect, the error being progressively larger with increasing L/d. Consequently, a new computer program was devised which yielded correct solutions independent of L/d. While the basic conclusions of the report remain unchanged, the corrected values of apparent hemispherical emittance, the critical performance parameter for honeycomb solar absorbers, are somewhat higher.

The attached corrected copies of figures 2 to 6, 8, and 9 and table III should be inserted in place of those presented in the report.

In addition, equation (11) on page 6 should read

$$dF_{X_o-X} = \left\{1 - \frac{|X - X_o|\left[2(X - X_o)^2 + 3\right]}{2\left[(X - X_o)^2 + 1\right]^{3/2}}\right\} dX$$

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TABLE III. - CALCULATED VALUES OF APPARENT HEMISPHERICAL EMITTANCE

[Wall thickness, 0.003 in. (0.00761 cm).]

Diameter of cylindrical cell, d		Cell length-to-diameter ratio, L/d	Base temperature, T_b							
			1860° R (1033 K)		2060° R (1144 K)					
			Wall material thermal conductivity, k							
			0.667 Btu/(hr)(in.)($^{\circ}$R) (0.138 W/(cm)(K))		0.667 Btu/(hr)(in.)($^{\circ}$R) (0.138 W/(cm)(K))		1.000 Btu/(hr)(in.)($^{\circ}$R) (0.208 W/(cm)(K))		1.333 Btu/(hr)(in.)($^{\circ}$R) (0.277 W/(cm)(K))	
in.	cm		Heat-conduction parameter, N_c	Apparent hemi-spherical emittance, $\bar{\epsilon}_a$	Heat-conduction parameter, N_c	Apparent hemi-spherical emittance, $\bar{\epsilon}_a$	Heat-conduction parameter, N_c	Apparent hemi-spherical emittance, $\bar{\epsilon}_a$	Heat conduction parameter, N_c	Apparent hemi-spherical emittance, $\bar{\epsilon}_a$
0.25	0.635	1	2.41	0.716	3.28	0.686	----	-----	----	-----
		3	↓	.404	↓	.378	----	-----	----	-----
		5	↓	.287	↓	.267	----	-----	----	-----
		7	↓	.225	↓	.208	----	-----	----	-----
0.5	1.27	1	9.65	0.602	13.1	0.586	----	-----	----	-----
		3	↓	.320	↓	.310	8.74	0.323	6.56	0.336
		5	↓	.222	↓	.215	8.74	.225	6.56	.234
		7	↓	.171	↓	.165	8.74	.173	6.56	.181
1.0	2.54	1	38.6	0.548	52.5	0.542	----	-----	----	-----
		3	↓	.290	↓	.287	----	-----	----	-----
		5	↓	.201	↓	.198	----	-----	----	-----
		7	↓	.154	↓	.152	----	-----	----	-----

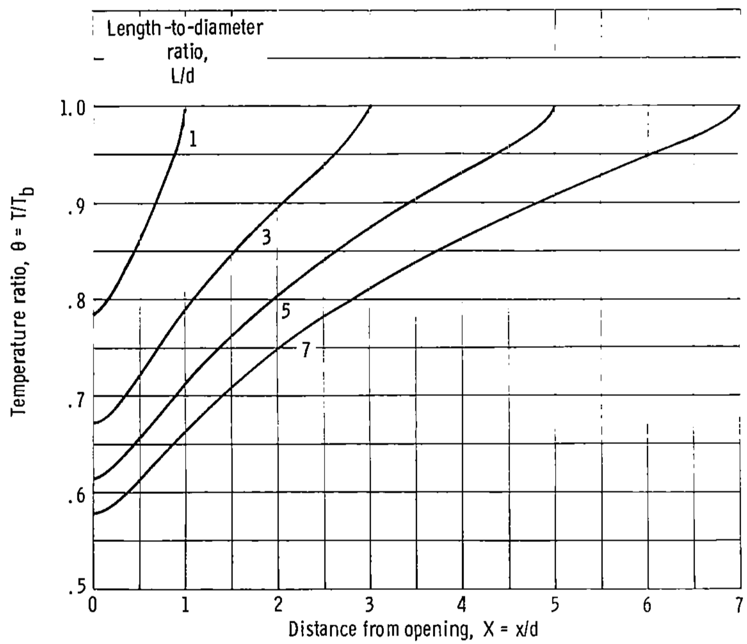


Figure 2. - Temperature ratio as function of distance from cell opening. Base temperature, 2060° R (1144 K); diameter, 0.5 inch (1.27 cm); thermal conductivity, 0.667 Btu per hour-inch-°R (0.1384 W/(cm)(K)); cell wall thickness, 0.003 inch (0.0076 cm).

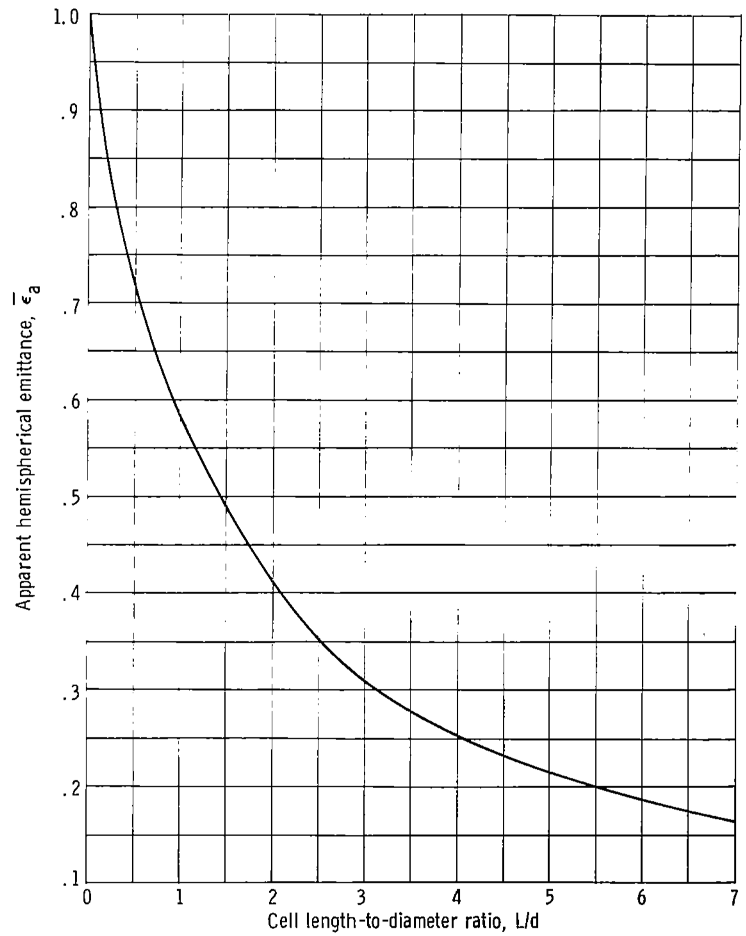


Figure 3. - Apparent hemispherical emittance as function of cell length-to-diameter ratio. Base temperature, 2060° R (1144 K); diameter, 0.5 inch (1.27 cm); thermal conductivity, 0.667 Btu per hour-inch-°R (0.138 W/(cm)(K)); cell wall thickness, 0.003 inch (0.00761 cm).

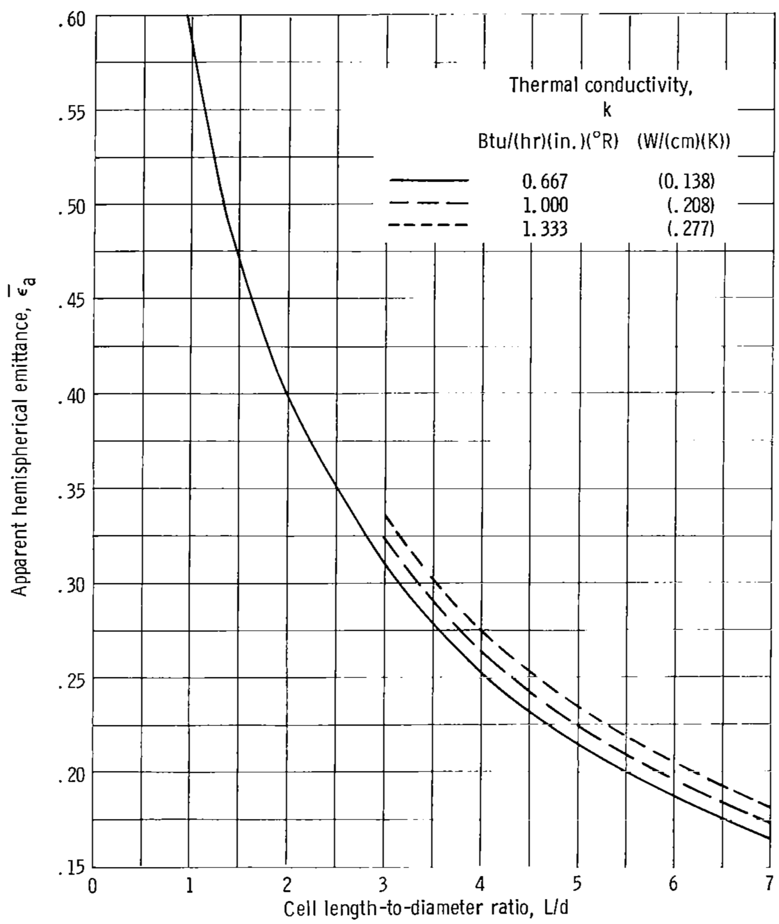
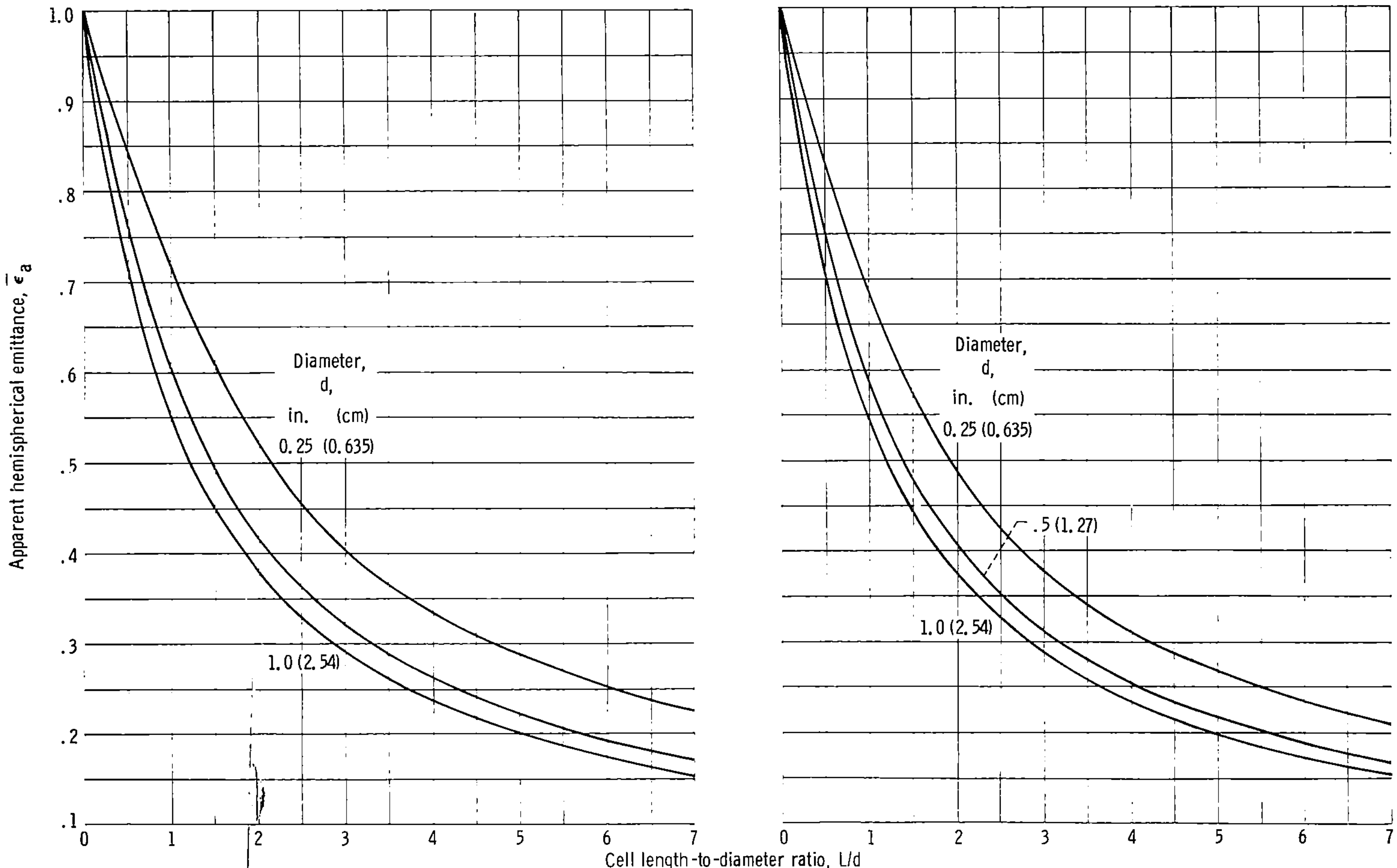


Figure 4. - Effect of thermal conductivity on apparent hemispherical emittance. Base temperature, 2060° R (1144 K); diameter, 0.5 inch (0.127 cm); cell thickness, 0.003 inch (0.00761 cm).



(a) Base temperature, 1860° R (1033 K).

(b) Base temperature, 2060° R (1144 K).

Figure 5. - Effect of cell diameter on apparent hemispherical emittance. Thermal conductivity, 0.667 Btu per hour-inch-°R (0.138 W/(cm)(K)); cell wall thickness, 0.003 inch (0.00761 cm).

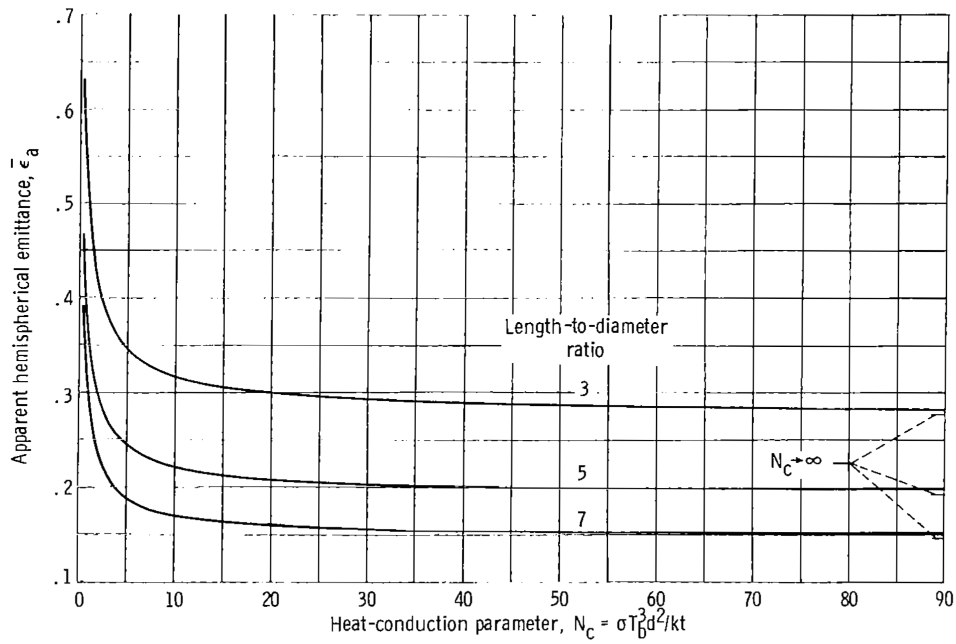


Figure 6. - Apparent hemispherical emittance as function of heat-conduction parameter.

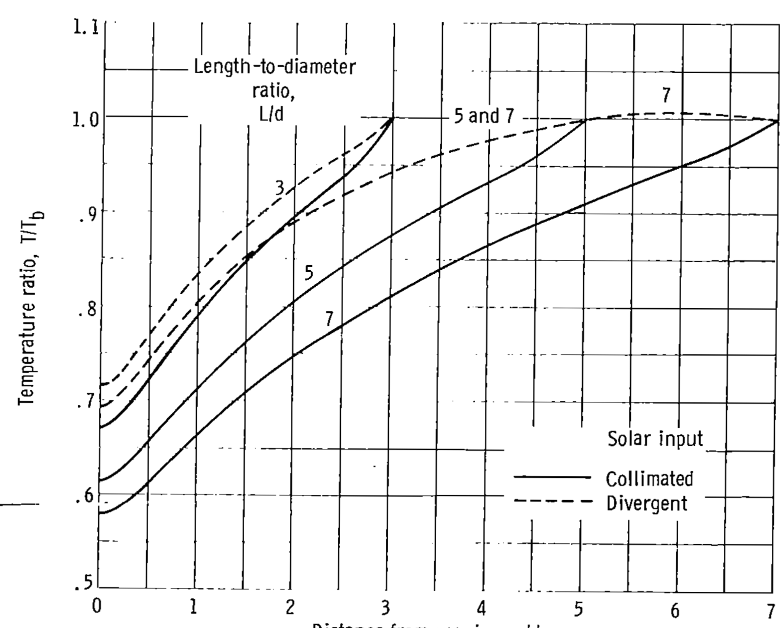


Figure 8. - Comparison of cell all temperature ratio for collimated and divergent solar input conditions for 0.. astronomical-unit distance from Sun. Reference conditions: base temper ıre, 2060° R (1144 K); diameter, 0.5 inch (1.27 cm); thermal conductivity, 0.667 Btu per hour-inch-°R (0.138 W/(cm)(K)); cell wall thickness, 0.003 inch (0.00761 cm).

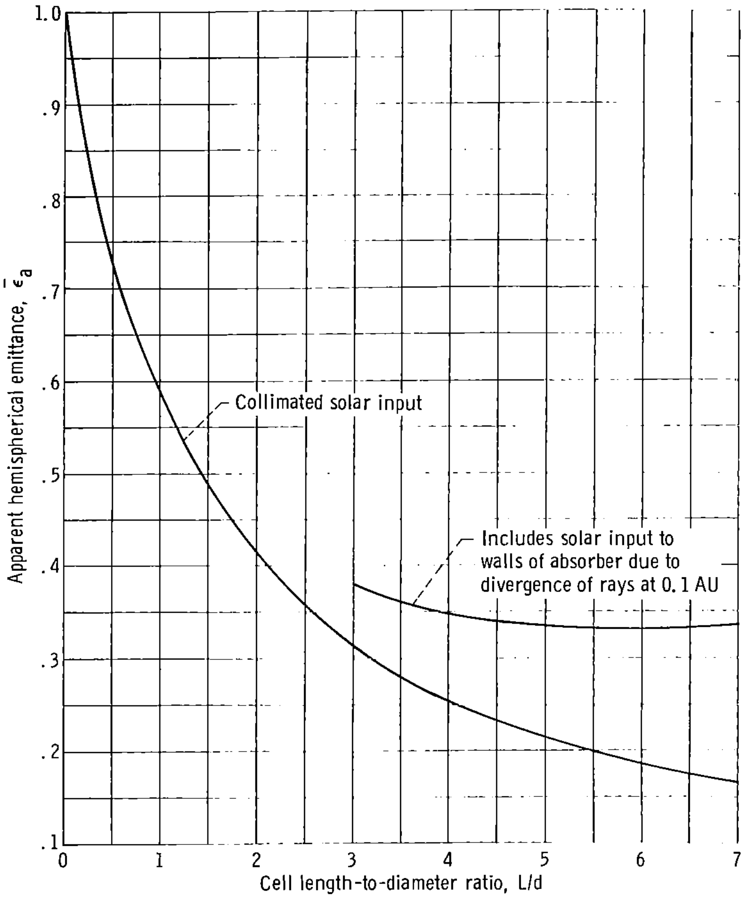


Figure 9. - Apparent hemispherical emittance as function of cell length-to-diameter ratio. Reference conditions: base temperature, 2060° R (1144 K); diameter, 0.5 inch (0.127 cm); thermal conductivity, 0.667 Btu per hour-inch-°R (0.138 W/(cm)(K)); cell wall thickness, 0.003 inch (0.00761 cm).

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ABSTRACT

A titanium alloy hexagonal honeycomb with blackened walls is considered as an absorber of collimated solar energy. Circular cylindrical geometry is assumed for the analysis as an approximation of the hexagonal cell structure. The apparent hemispherical emittance of such an absorber positioned over a black surface is calculated. Results are presented for cell length-to-diameter ratios of from 1 to 7 and cell diameters of 0.25, 0.5, and 1.0 in. (0.635, 1.27, and 2.54 cm, respectively). The collimated incident solar flux is assumed sufficient to attain base surface temperatures of 1860° and 2060° R (1033 and 1144 K). The corresponding weight per unit area of such an absorber is also calculated.

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SUMMARY

A titanium alloy hexagonal honeycomb with blackened walls was considered as an absorber of collimated solar energy. Circular cylindrical geometry was assumed as an approximation of the hexagonal cell structure for the analysis. Calculated values of the apparent hemispherical emittance were presented for base surface temperatures of 1860° and 2060° R (1033 and 1144 K), cell length-to-diameter ratios from 1 to 7, and cell diameters of 0.25, 0.5, and 1.0 inch (0.635, 1.27, and 2.54 cm, respectively).

The results indicated that for a titanium alloy honeycomb an apparent hemispherical emittance of the order of 0.1 can be achieved with a cell diameter of 0.5 inch (1.27 cm) and a length-to-diameter ratio of about 6. The calculated weight per unit area of such an absorber fabricated from 0.006 inch (0.01523 cm) titanium foil is about 1.65 pounds per square foot (0.81 g/cm^2), excluding the absorber base.

INTRODUCTION

Thermoelectric, thermionic, and Brayton- and Rankine-cycle turbine generators are examples of heat engines that could be used to provide auxiliary electrical power aboard future spacecraft. Such heat engines, using solar radiation as a source of energy, would require receivers capable of absorbing and retaining a large fraction of the incident solar flux for conversion to electrical energy. Because the efficiency of a heat engine is dependent on the temperature difference between the hot reservoir (receiver) and the cold reservoir (radiator), it is obviously desirable to operate at the highest possible receiver temperature consistent with thermal reradiation loss and material stability considerations.

One method of utilizing solar energy is to collect the radiation with a large curved reflector and focus it into a cavity absorber. Another method of absorbing and retaining a large fraction of the incident solar radiation is to use a selective coating on the absorber surface. Such a coating would have a high absorptance for solar radiation (short wavelength region) and a low total emittance in the infrared region. A number of coatings having these properties have been tested (ref. 1); however, they have not yet been subjected to long-term testing at temperatures above about 1460° R (811 K). In addition, the performance of such absorber coatings is known to decrease with increasing temperature.

A third technique for absorbing solar energy at high temperatures (above 1460° R or 811 K) is to place a honeycomb structure over the absorber surface, which would allow incident collimated solar energy to pass through to the absorber surface while limiting (attenuating) the reradiation of thermal energy from the absorber surface. Such a honeycomb structure would be fabricated from a material having a low thermal conductivity to minimize the conductive losses along the honeycomb wall (and subsequent radiation to space) and a high strength-to-weight ratio at elevated temperatures.

Although the performance of honeycombs fabricated from fragile materials such as glass has been investigated for terrestrial use at temperatures below 1460° R (811 K) (refs. 2 and 3), no information was found in the literature for high-temperature honeycombs suitable for space application. This study therefore considers the performance of a titanium-foil honeycomb structure consisting of close-packed cells that are hexagonal in cross section. The cell walls as well as the base surface on which the honeycomb rests are assumed black, that is, all incident energy is absorbed. The honeycomb structure is assumed to be oriented with respect to the collimated incident solar rays such that the cell walls receive no solar radiation. (The effect of the divergence of the solar rays is considered in the appendixes for one specific case at 0.1 AU from the Sun.) The use of these assumptions results in a solar absorptance of 1; hence, only the radiant energy emitted from the honeycomb must be determined to characterize its performance. The ratio of the radiant flux per unit area emitted from a honeycomb at base temperature T_b to that of a blackbody radiator at the same temperature is defined as the apparent hemispherical emittance $\bar{\epsilon}_a$.

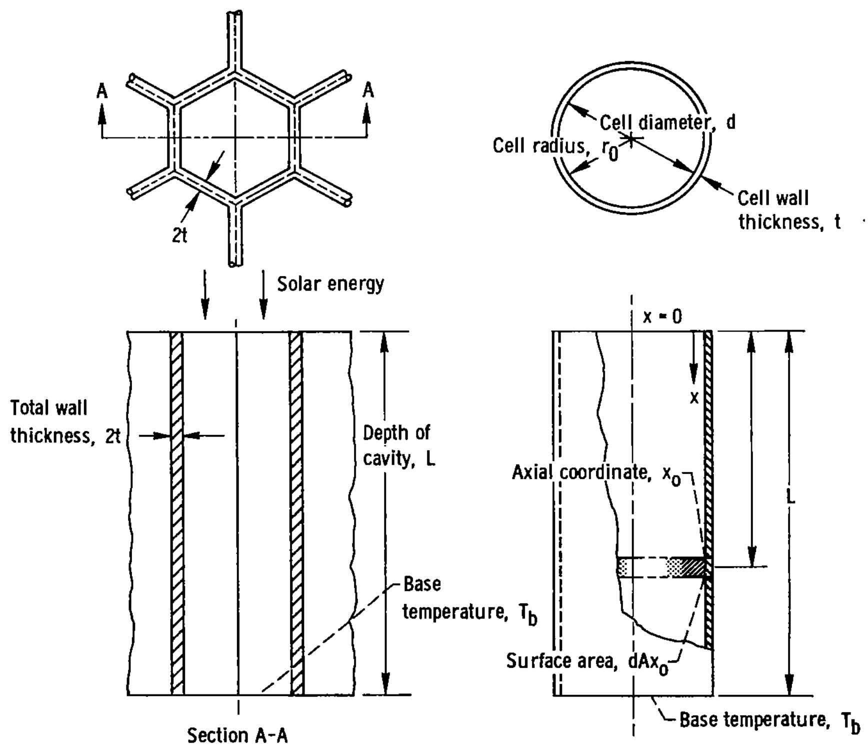
Calculated values of apparent hemispherical emittance are presented for base surface temperatures of 1860° and 2060° R (1033 and 1144 K), cell length-to-diameter ratios of 1, 3, 5, and 7, cell diameters of 0.25, 0.5, and 1.0 inch (0.635, 1.27, and 2.54 cm, respectively), and a cell wall thickness of 0.003 inch or 0.00761 centimeters (total wall thickness of 0.006 in. or 0.01523 cm). The wall material thermal conductivity was fixed at 0.667 Btu per hour-inch-°R (0.1384 W/(cm)(K)) for most calculations, although the effect of increasing the value to 1.0 and 1.33 Btu per hour-inch-°R (0.208 and 0.277 W/(cm)(K)) was also included for a representative case.

The weight-to-area ratio of a titanium (Ti)-6 aluminum (Al)-4 vanadium (V) alloy honeycomb structure, excluding the absorber base, is presented as a function of the cell length-to-diameter ratio for cell wall thicknesses of 0.003 and 0.001 inch (0.00761 and 0.00254 cm). Additional information is also presented for the calculation of the apparent emittance for other suitable materials and/or cell dimensions, that is, for other combinations of thermal conductivity, cell diameter, cell wall thickness, and absorber surface temperature.

METHOD OF CALCULATION

Description of Model

For the determination of the radiant energy emitted from a cell of a hexagonal honeycomb (fig. 1(a)) for a given base temperature T_b, the radiation emitted from both the base and the walls of the cell must be considered. The analysis was simplified by using a circular cylindrical cell with a diameter equal to the hexagonal cell minimum diameter (as shown in fig. 1(b)) to approximate the hexagonal cell. The cylindrical cell



(a) Hexagonal cell structure.

(b) Circular cylindrical cell used in analysis. (Approximation of hexagonal structure).

Figure 1. - Unit cell of honeycomb solar absorber.

has a length L, a diameter d, a radius r_0, and a wall thickness t, which represents one-half the wall thickness of the close-packed hexagonal cells. Both the wall and base surfaces are assumed to be black (i.e., all incident energy is absorbed), while the exposed edge of the cell wall is assumed to be perfectly reflecting.

Procedure

The analysis of such a circular cylindrical cell having nonisothermal walls is presented in reference 4 for the case of an external diffuse-radiation source and fixed uniform base temperatures. Since the case considered herein assumed a cylinder oriented with respect to perfectly collimated radiation, the analysis of reference 4 had to be modified. The modification reduced the problem to one in which the cylinder was considered independently of the external radiation; that is, the cylinder wall was assumed to receive no incident solar radiation. Again, as in reference 4, base surface temperatures were arbitrarily chosen by assuming that a heat balance existed between the base and the heat engine. The modified form of the analysis of reference 4 is presented in the remainder of this section.

From figure 1(b), a typical cylinder wall element located at $x = x_o$ has a surface area $dA_{x_0} = (\pi d)dx_o$. (Symbols are defined in appendix A.) The energy balance on such an element is as follows:

$$\left(dQ_{net}\right)_{rad} + \left(dQ_{net}\right)_{cond} = 0 \tag{1}$$

If the temperature difference across the wall thickness is assumed to be negligible, the conduction term is

$$\left(dQ_{net}\right)_{cond} = -k(\pi d)t \frac{d^2T}{dx^2} dx_o \tag{2}$$

where the thermal conductivity of the wall is assumed independent of temperature, and $(\pi d)t$ approximates the wall cross-sectional area.

In the determination of the net radiation $\left(dQ_{net}\right)_{rad}$, all incident energy is assumed to be absorbed (i.e., surfaces are black). The net radiation is therefore equal to the difference between the energy emitted by the element and the energy incident upon it. The energy emitted from the element, given by the Stefan-Boltzmann law, is

$$\epsilon\sigma T^4(x_o)dA_{x_o} \tag{3}$$

where $\epsilon = 1$ for a black surface. The incident radiation consists of contributions from both the cylindrical and base surfaces of the cavity. The radiation emitted by an element dA_x on the cylinder wall and absorbed by an element dA_{x_o} is

$$\sigma T^4_{(x)} \, dF_{x-x_o} \, dA_x \tag{4}$$

where dF_{x-x_o} is the diffuse-angle factor between an element dA_x and an element dA_{x_o}. The reciprocity theorem for diffuse-angle factors states that $dF_{x-x_o} \, dA_x = dF_{x_o-x} dA_{x_o}$. Therefore, expression (4) becomes

$$\sigma T^4(x) dF_{x_o-x} \, dA_{x_o} \tag{5}$$

The total radiation from the cylinder wall to the element dA_{x_o} is found by integration:

$$dA_{x_o} \int_{x=0}^{L} \sigma T^4(x) dF_{x_o-x} \tag{6}$$

The radiation from the base surface to the element dA_{x_o}, found in a similar manner, is

$$\sigma T_b^4 F_{x_o-b} \, dA_{x_o} \tag{7}$$

Terms (6) and (7) are then subtracted from (3) to give the net radiation

$$\left(dQ_{net}\right)_{rad} = \left[\sigma T^4_{(x_o)} - \int_{x=0}^{L} \sigma T^4(x) dF_{x_o-x} - \sigma T_b^4 F_{x_o-b}\right] dA_{x_o} \tag{8}$$

The energy balance equation (1) can now be evaluated. Substituting equations (2) and (8) into equation (1) and introducing dimensionless variables yield

$$\frac{d^2\theta}{dX_o^2} = N_c \left[\theta^4 - \int_{X=0}^{L/d} \theta^4(X) dF_{X_o-X} - F_{X_o-b}\right] \tag{9}$$

where

$$X_o = \frac{x_o}{d}$$

$$X = \frac{x}{d}$$

$$\theta = \frac{T}{T_b}$$

$$N_c = \frac{\sigma T_b^3 d^2}{kt}$$

The parameter N_c is herein defined as the heat conduction parameter. Note that, for given values of the base temperature and cylinder dimensions, N_c is solely dependent on the thermal conductivity of the wall material.

Equation (9) is a nonlinear integro-differential equation from which the axial temperature distribution θ along the cylinder wall can be calculated. A method similar to that used in reference 5 was used to solve this equation. The boundary conditions and angle factors are now defined.

If the exposed edge of the cylinder wall is assumed to be perfectly reflecting, then at $x = 0$, $dT/dx = 0$. At $x = L$, the temperature T is set equal to T_b. The boundary conditions are thus given by equations (10) as

$$\frac{d\theta}{dX} = 0 \qquad \text{at } X = 0 \tag{10a}$$

$$\theta = 1 \qquad \text{at } X = \frac{L}{d} \tag{10b}$$

The expressions for the angle factors, derived from angle-factor algebra, are

$$dF_{X_o-X} = \left\{1 - \frac{|X - X_o| \left[2(X - X_o)^3 + 3\right]}{2\left[(X - X_o)^2 + 1\right]^{3/2}}\right\} dX \tag{11}$$

$$F_{X_o-b} = \left\{ \frac{1}{2} \frac{1 + 2\left(\frac{L}{d} - X_o\right)^2}{\left[\left(\frac{L}{d} - X_o\right)^2 + 1\right]^{1/2}} - \left(\frac{L}{d} - X_o\right) \right\} \tag{12}$$

Both equations (11) and (12) were obtained from reference 4.

With a solution determined for the axial temperature distribution (eq. (9)), the amount of energy radiated from the cylindrical cavity can be calculated. From a given element dA_x, the energy radiated out through the cavity opening is

$$\sigma T^4(x) dA_x \, F_{x-o} \tag{13}$$

The total amount of radiated flux emitted through the cavity opening from the cylinder wall is found by integration

$$q_w = 4\sigma T_b^4 \int_0^{L/d} \theta^4(X) F_{X-o} \, dX \tag{14}$$

where q_w is the rate at which energy is radiated out of the cavity from the cylinder wall per unit area of the opening, and F_{X-o} is the diffuse-angle factor between an element at $X = X_o$ and a disk at $X = 0$. Adding the radiated flux emitted from the base surface to the flux from the wall gives the total rate at which energy is radiated out of the cavity per unit area of opening q_t

$$q_t = \sigma T_b^4 \left[4 \int_0^{L/d} \theta^4(X) F_{X-o} \, dX + F_{b-o} \right] \tag{15}$$

where F_{b-o} is the diffuse-angle factor between a disk at $X = L/d$ (i.e., $T = T_b$) and $X = 0$. The expressions for the angle factors are given as

$$F_{X-o} = \frac{1}{2} \frac{1 + 2X^2}{\left(X^2 + 1\right)^{1/2}} - X \qquad \text{(ref. 4)} \tag{16}$$

$$F_{b-o} = 1 + 2\left(\frac{L}{d}\right)^2 - 2\left(\frac{L}{d}\right)\sqrt{\left(\frac{L}{d}\right)^2 + 1} \qquad \text{(ref. 6)} \tag{17}$$

TABLE I. - VALUES OF HEAT-CONDUCTION PARAMETER FOR WHICH APPARENT HEMISPHERICAL EMITTANCE IS CALCULATED AT LENGTH-TO-DIAMETER RATIOS OF 1, 3, 5 AND 7.

[Thickness, t, 0.003 in. (0.00761 cm).]

Base temperature, T_b		Thermal conductivity, k		Diameter of cylindrical cell, d, in. (cm)		
		Btu/(hr)(in.)(^{o}R)	W/(cm)(K)	0.25 (0.635)	0.5 (1.27)	1.0 (2.54)
^{o}R	K			Heat-conduction parameter, N_c		
1860	1033	0.667	0.1384	2.413	9.652	38.609
2060	1144	.667	.1384	3.278	13.112	52.45
2060	1144	1.000	.208	-----	[a]8.742	------
2060	1144	1.333	.277	-----	[a]6.556	------

[a]Evaluated for L/d values of 3, 5, and 7 only.

Thus, for given values of N_c, L/d, and T_b, it is possible to calculate the apparent hemispherical emittance $\bar{\epsilon}_a$ of the cavity, where

$$\bar{\epsilon}_a = \frac{q_t}{\sigma T_b^4} = 4 \int_0^{L/d} \theta^4(X) F_{X-o}\, dX + F_{b-o} \tag{18}$$

With the use of a digital computer, numerical solutions of equations (9) and (18) were obtained for the values of N_c and L/d shown in table I. Although the indicated values of the conduction parameter N_c were determined for preselected values of T_b, d, k, and t given in table I, the results are equally valid for other combinations of T_b, L, d, k, and t yielding the given values of N_c and L/d.

RESULTS AND DISCUSSION

Apparent Hemispherical Emittance

The effect of varying the cell length-to-diameter ratio L/d on the axial temperature distribution is illustrated in figure 2, where the temperature ratio T/T_b is presented as a function of the distance from the opening x/d for the following reference case: N_c = 13.112 (T_b = 2060^{o} R or 1144 K); d = 0.5 inch (1.27 cm); k = 0.667 Btu per hour-inch-^{o}R (0.1384 W/(cm)(K)); and t = 0.003 inch (0.0076 cm)). Note that for an L/d of 5, for example, the wall temperature at the cell opening is only about half

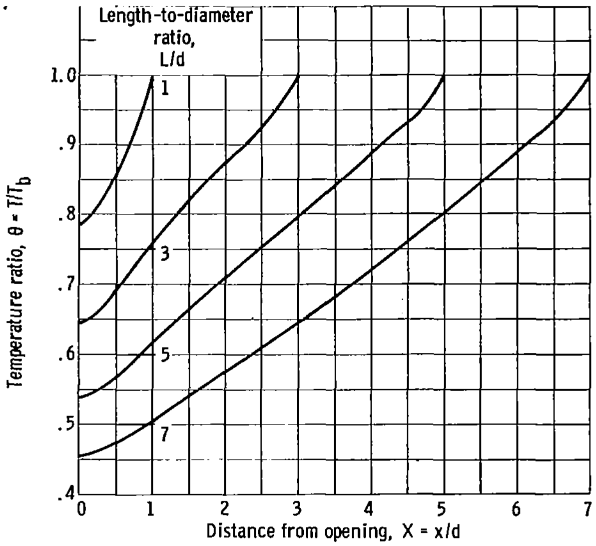


Figure 2. - Temperature ratio as function of distance from cell opening. Base temperature, 2060° R (1144 K); diameter, 0.5 inch (1.27 cm); thermal conductivity, 0.667 Btu per hour-inch-°R (0.1384 W/(cm)(K)); cell wall thickness, 0.003 inch (0.0076 cm).

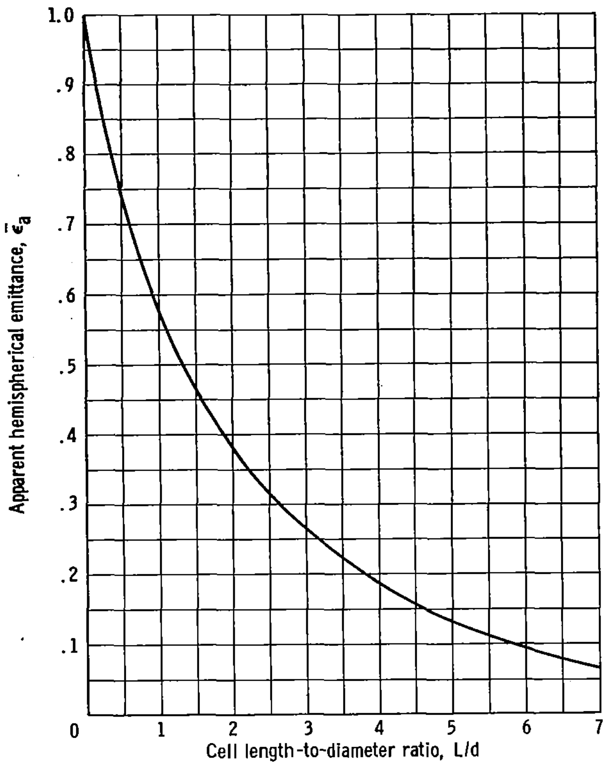


Figure 3. - Apparent hemispherical emittance as function of cell length-to-diameter ratio. Base temperature, 2060° R (1144 K); diameter, 0.5 inch (1.27 cm); thermal conductivity, 0.667 Btu per hour-inch-°R (0.1384 W/(cm)(K)); cell wall thickness, 0.003 inch (0.00761 cm).

the base temperature. The effect of this temperature decrease in the region near the cell opening is to lower the apparent hemispherical emittance $\bar{\epsilon}_a$, as shown in figure 3. At an L/d of 5, $\bar{\epsilon}_a$ is 0.13, and increasing L/d from 5 to 7 results in a decrease in $\bar{\epsilon}_a$ to 0.065.

The thermal conductivity k of the Ti-6Al-4V alloy, obtained from reference 7, is presented in table II for various temperatures. Since the analysis assumed k to be independent of temperature, a reference value of 0.667 Btu per hour-inch-°R (0.1384 W/(cm)(K)) was used initially. Also, $\bar{\epsilon}_a$ was evaluated for values of k of 1.00 and 1.33 Btu per hour-inch-°R (0.208 and 0.277 W/(cm)(K)) and compared with the reference condition in figure 4 to assess the effect of variations in the wall material thermal conductivity. For an L/d of 5, for example, $\bar{\epsilon}_a$ increases from 0.130 to 0.155 when k is doubled from 0.667 to 1.333 Btu per hour-inch-°R (0.1384 to 0.277 W/(cm)(K)). This effect of increasing conductivity might be offset by halving the wall thickness.

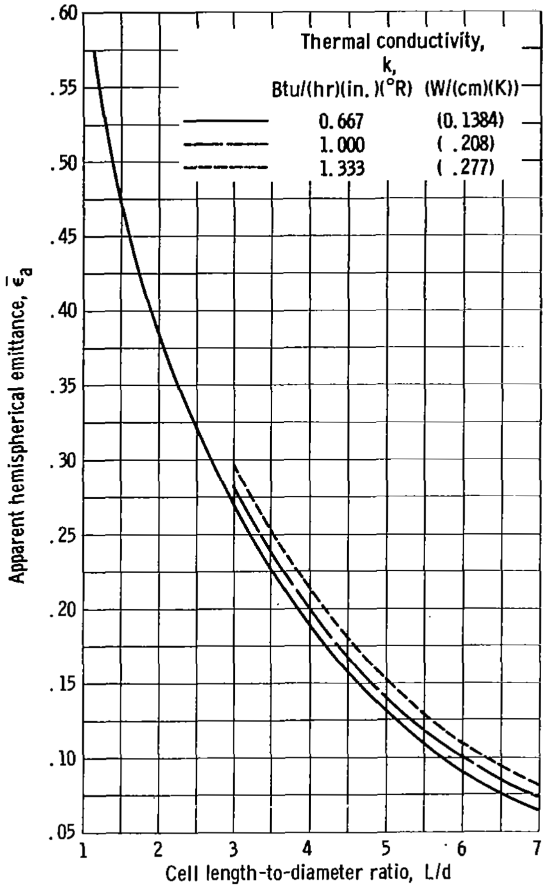
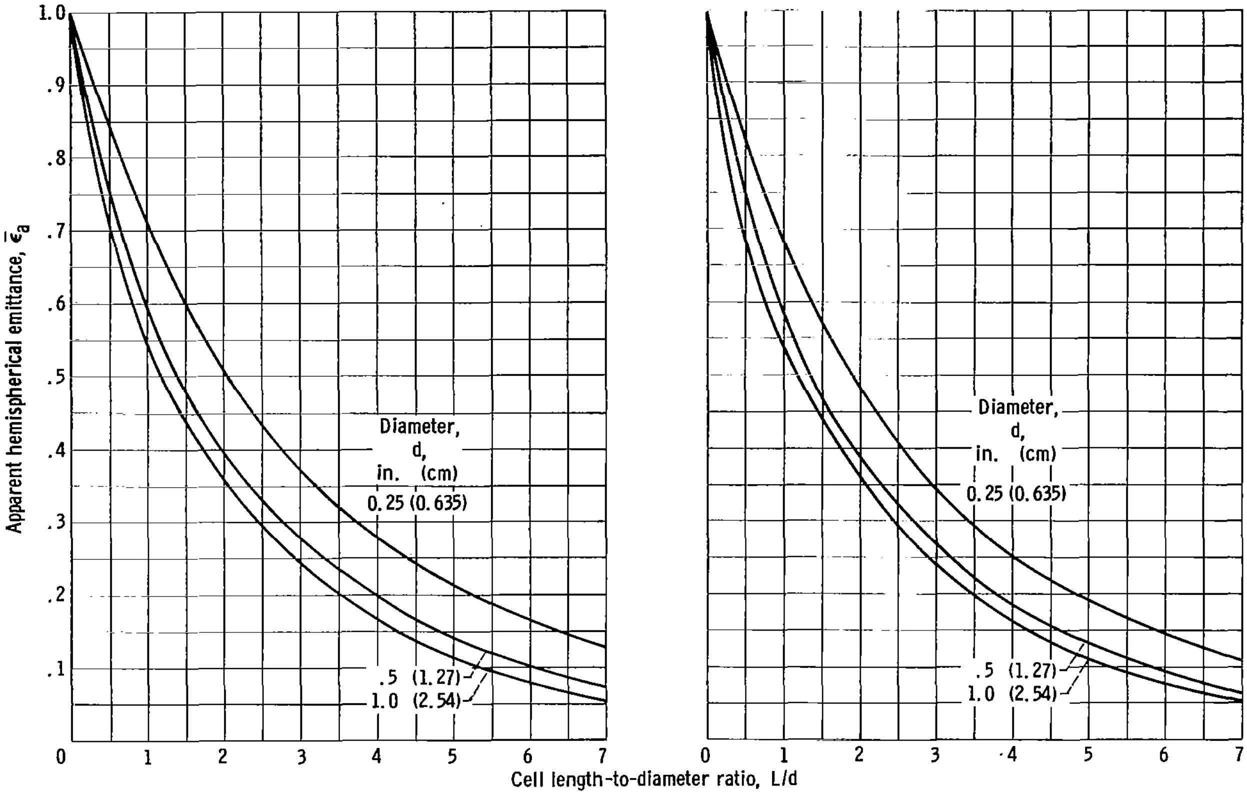


Figure 4. - Effect of thermal conductivity on apparent hemispherical emittance. Base temperature, 2060° R (1144 K); diameter, 0.5 inch (1.27 cm); cell thickness, 0.003 inch (0.00761 cm).

TABLE II. - THERMAL CONDUCTIVITY OF SHEET TITANIUM - 6-ALUMINUM-4VANADIUM ALLOY

[Data from ref. 7.]

Temperature		Thermal conductivity, k	
°R	K	Btu/(hr)(in.)(°R)	W/(cm)(K)
672	373	0.417	0.087
860	477	.516	.107
1260	700	.716	.149
1660	922	.916	.190



(a) Base temperature, 1860° R (1033 K).

(b) Base temperature, 2060° R (1144 K).

Figure 5. - Effect of cell diameter on apparent hemispherical emittance. Thermal conductivity, 0.667 Btu per hour-inch-°R (0.1384 W/(cm)(K)); cell wall thickness, 0.003 inch (0.00761 cm).

The apparent hemispherical emittance $\bar{\epsilon}_a$ is presented in figures 5(a) and (b) for base temperatures T_b of 1860^o and 2060^o R (1033 and 1144 K). The decrease in $\bar{\epsilon}_a$ with increasing cell diameter d for a given L/d results from the attendant increase in cell length and thus from the reduced heat conduction. For cell diameters in the range of 0.5 to 1.0 inch (1.27 to 2.54 cm), $\bar{\epsilon}_a$'s of the order of 0.1 result for L/d's of 6 or more. In comparison, for an ideal selective solar absorber coating, as defined in reference 8, the theoretical hemispherical emittance is 0.03, and solar absorptance is 0.87 for a surface temperature of 2060^o R (1144 K) and a solar concentration of 10 (ref. 8). However, such coatings for use at this temperature have not been developed.

Values of the apparent hemispherical emittance of a cell, evaluated for a range of N_c of 0 to 60, are presented in figure 6. Any combination of parameters T_b, d, k, and t resulting in N_c's in this range can be used to determine $\bar{\epsilon}_a$ for a given L/d, as will be illustrated in the section Weight-to-Area Ratio of Honeycomb.

TABLE III. - CALCULATED VALUES OF APPARENT HEMISPHERICAL EMITTANCE

[Wall thickness, 0.003 in. (0.00761 cm).]

Diameter of cylindrical cell, d		Cell length-to-diameter ratio, L/d	Base temperature, T_b							
			1860° R (1033 K)		2060° R (1144 K)					
			Wall material thermal conductivity, k							
			0.667 Btu/(hr)(in.)($^\circ$R) (0.1384 W/(cm)(K))		0.667 Btu/(hr)(in.)($^\circ$R) (0.1384 W/(cm)(K))		1.000 Btu/(hr)(in.)($^\circ$R) (0.208 W/(cm)(K))		1.333 Btu/(hr)(in.)($^\circ$R) (0.277 W/(cm)(K))	
in.	cm		Heat-conduction parameter, N_c	Apparent hemi-spherical emittance, $\overline{\epsilon}_a$	Heat-conduction parameter, N_c	Apparent hemi-spherical emittance, $\overline{\epsilon}_a$	Heat-conduction parameter, N_c	Apparent hemi-spherical emittance, $\overline{\epsilon}_a$	Heat conduction parameter, N_c	Apparent hemi-spherical emittance, $\overline{\epsilon}_a$
0.25	0.635	1	2.413	0.715	3.278	0.684	-----	-----	-----	-----
		3	↓	.372	↓	.344	-----	-----	-----	-----
		5	↓	.212	↓	.189	-----	-----	-----	-----
		7	↓	.125	↓	.107	-----	-----	-----	-----
0.50	1.27	1	9.652	0.599	13.112	0.583	-----	-----	-----	-----
		3	↓	.279	↓	.268	8.742	0.283	6.556	0.297
		5	↓	.139	↓	.131	8.742	.142	6.556	.153
		7	↓	.070	↓	.064	8.742	.073	6.556	.080
1.0	2.54	1	38.609	0.545	52.45	0.539	-----	-----	-----	-----
		3	↓	.245	↓	.241	-----	-----	-----	-----
		5	↓	.113	↓	.110	-----	-----	-----	-----
		7	↓	.052	↓	.050	-----	-----	-----	-----

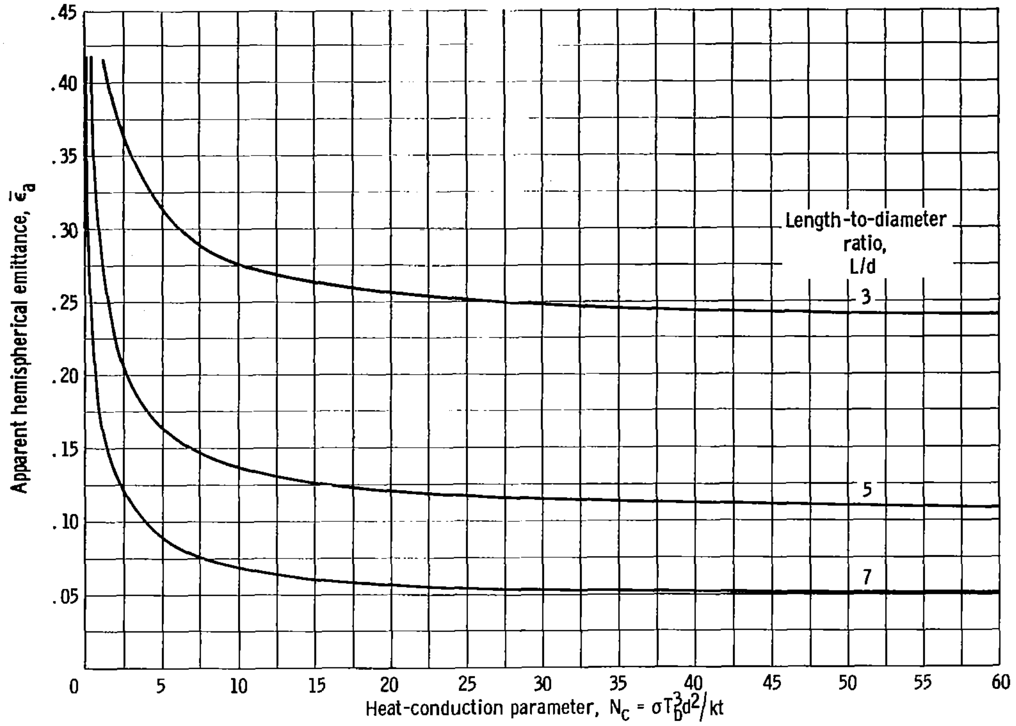


Figure 6. - Apparent hemispherical emittance as function of heat-conduction parameter.

Calculated values of $\bar{\epsilon}_a$ for the specified conditions of T_b, d, k, and t are presented in table III.

Absorber Efficiency

In this analysis, the apparent hemispherical emittance is used to characterize the performance of a honeycomb solar absorber. Generally, however, the determination of the amount of usable solar energy made available by such an absorber is of more interest. The efficiency of the absorber η is taken as the ratio of the usable solar heat per unit area to that incident so that the absorber performance can be determined as a function of the distance from the Sun as follows:

$$\eta = \frac{443\ S^{-2} - \bar{\epsilon}_a \sigma T_b^4}{443^{S-2}}$$

where

$$\sigma = 1.73\times10^{-9} \text{ Btu per hour-square foot-}{}^{\circ}\text{R}^4$$

TABLE IV. - ABSORBER EFFICIENCY FOR APPARENT

HEMISPHERICAL EMITTANCE OF 0.1 AND BASE

TEMPERATURE OF 1860° R (1033 K)

Distance from Sun, s, AU	Absorber efficiency, η, percent	Usable solar heat per unit area	
		Btu/(hr)(ft^2)	W/m^2
1.000	0	0	0
.465	0	0	0
.400	.26	720	2 280
.300	.583	2 870	9 070
.200	.816	9 050	28 600
.100	.955	42 250	133 800

or

$$\eta = \frac{0.14\ s^{-2} - \bar{\epsilon}_a \sigma T_b^4}{0.14\ s^{-2}}$$

where

$$\sigma = 5.67\times10^{-12} \text{ watt per square centimeter - K}^4.$$

Note that s is the distance from the Sun in astronomical units for both expressions, and a solar absorptance of 1 has been assumed. For the case of $\bar{\epsilon}_a = 0.1$ and $T_b = 1860^{\circ}$ R (1033 K), the absorber efficiency as a function of the distance from the Sun is given in table IV. Note that the absorber efficiency is 0 from 1.0 to 0.465 astronomical unit, whereas from 0.3 to 0.1 astronomical unit, it increases sharply from 0.583 to 0.955. Hence, such a honeycomb absorber operating under the given conditions might be useful in the vicinity of Mercury (0.388 AU) or at distances closer to the Sun.

Weight-to-Area Ratio of Honeycomb

The weight per unit area of a hexagonal honeycomb structure, excluding the absorber base, is shown in figure 7 for Ti -6Al-4V alloy wall thicknesses t of 0.001 and 0.003 inch (0.00254 and 0.00761 cm). The physical density of the Ti - 6Al-4V alloy is 0.16 pound per cubic inch (4.47 g/cu cm) (ref. 7). For an L/d of 6, for example, the weight-to-area ratio is about 1.65 pounds per square foot (0.81 g/cm^2) for

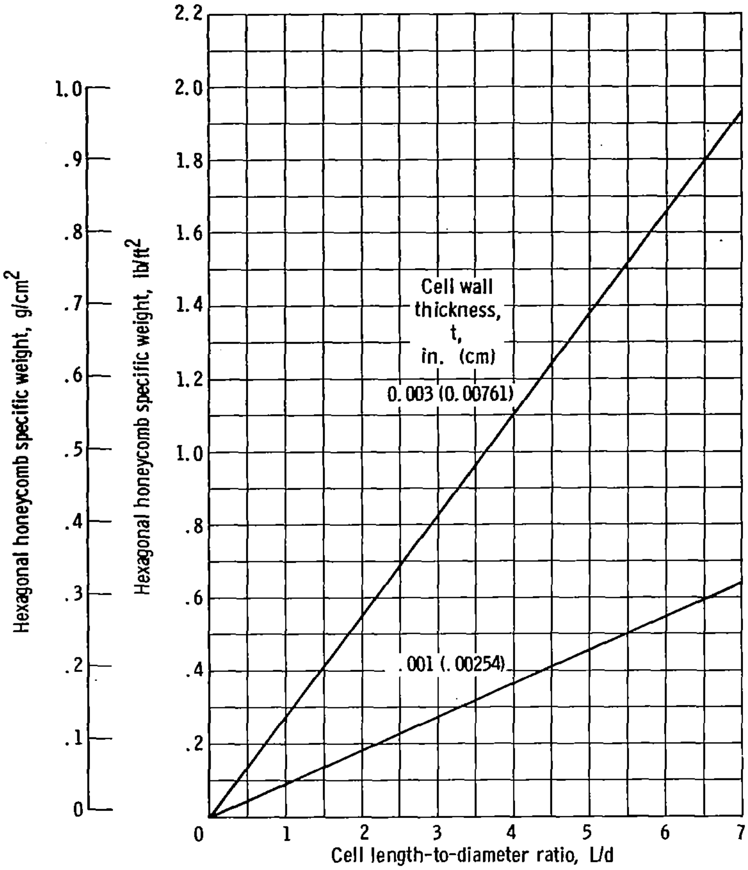


Figure 7. - Hexagonal honeycomb solar absorber specific weight as function of cell length-to-diameter ratio for 6Al-4V titanium alloy with density of 0.16 pound per cubic inch (4.47 g/cu cm).

$t = 0.003$ inch (0.00761 cm). Reducing t to 0.001 inch (0.00254 cm) at the same L/d results in a proportionate decrease in weight to 0.55 pound per square foot (0.27 g/cm^2).

Titanium (6Al-4V) alloy honeycombs with a total wall thickness $2t$ as small as 0.002 inch (0.00508 cm) have actually been fabricated, as reported in reference 9. Since the $\bar{\epsilon}_a$ values were calculated for N_c values corresponding to a wall thickness t of 0.003 inch (0.00761 cm), figure 6 must be used to determine the effect on $\bar{\epsilon}_a$ of reducing the wall thickness (changing N_c). For example, at an L/d of 7 and an N_c of 3.278 (when $T_b = 2060^{\circ}$ R or 1144 K; $d = 0.25$ in. or 0.635 cm; $k = 0.667$ Btu/(hr)(in.)($^{\circ}$R) or 0.1324 W/(cm)(K); and $t = 0.003$ in. or 0.0761 cm), the apparent hemispherical emittance $\bar{\epsilon}_a$ is 0.106 in figure 5(b). Reducing the wall thickness to 0.001 inch (0.00254 cm), that is, increasing N_c to 9.834, results in a decrease in $\bar{\epsilon}_a$ to 0.07 as shown in figure 6.

CONCLUDING REMARKS

The analysis of the absorber performance presented in this report assumed a honeycomb structure perfectly oriented with respect to collimated solar radiation. Hence the effects of misorientation and the divergence of solar radiation should be considered for practical applications. Furthermore, a zero temperature gradient at the cell opening was used as a boundary condition by assuming that the exposed edge of the cylinder wall was perfectly reflecting. However, for certain cases, such as a distance of 0.2 astronomical unit or less from the Sun, the incident solar radiation absorbed at the cell edge would result in emittance values higher than those calculated. For example, if only the effect of the divergent radiation on the cell walls at 0.1 astronomical unit from the Sun were considered for a length-to-diameter ratio of 5.75 at reference conditions, the apparent hemispherical emittance $\bar{\epsilon}_a$ would increase to 0.19 as compared with an $\bar{\epsilon}_a$ of 0.1 for collimated solar radiation (see calculations in appendix C).

SUMMARY OF RESULTS

A preliminary analysis of a titanium honeycomb solar absorber, with blackened walls positioned over a black surface, was performed for base temperatures of 1860° and 2060° R (1033 and 1144 K). Perfect orientation and collimated solar radiation were assumed. The study yielded the following results:

1. With the incident solar flux assumed sufficient to obtain absorber surface temperatures of 1860° R (1033 K) or above, apparent hemispherical emittances of less than 0.1 were indicated for values of length-to-diameter ratios greater than 6.

2. A hexagonal close-packed cell structure was assumed for the honeycomb absorber fabricated from titanium - 6-aluminum-4-vanadium alloy foil 0.006-inch (0.0152 cm) thick. The weight-to-area ratio of such an absorber with a length-to-diameter ratio of 6, excluding the absorber base, is about 1.65 pounds per square foot (0.81 g/cm^2). With the use of a foil thickness (2t) of 0.002 inch (0.00508 cm), from which honeycombs have been fabricated, the weight can be reduced to 0.55 pound per square foot (0.27 g/cm^2).

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, May 23, 1968,
120-27-06-06-22.

APPENDIX A

SYMBOLS

A	surface area
d	diameter of cylindrical cell
F	diffuse-angle factor
h	height
k	thermal conductivity
L	depth of cell
N_c	heat-conduction parameter
Q	total heat-transfer rate
q	heat-transfer rate per unit area
r	radius
r_0	radius of cylindrical cell
S	solar radiation
s	distance coordinate
T	absolute temperature
t	cell wall thickness
X	dimensionless axial coordinate
x	axial coordinate
α	angle
β	angle
ϵ	hemispherical emittance
η	absorber efficiency
θ	dimensionless temperature T/T_b
ρ	variable radial coordinate
σ	Stefan-Boltzmann constant
ω	solid angle

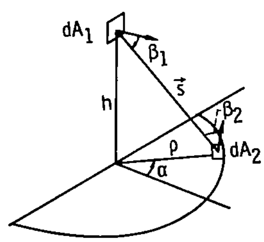
Subscripts:

a	apparent
b	base surface
cond	conduction
net	net
o	opening of cavity
rad	radiation
s	solar divergent rays
tot	total
x	axial coordinate
w	wall

APPENDIX B

ESTIMATE OF SOLAR INPUT TO WALLS OF BLACK HONEYCOMB SOLAR ABSORBER AT 0.1 ASTRONOMICAL UNIT FROM SUN

Herein, the apparent hemispherical emittance $\bar{\epsilon}_a$ of a circular cylindrical cell with blackened walls was calculated with the assumption of perfectly collimated solar input to simplify the analysis. For the analysis, only radiant energy contributions from both the base and wall surfaces of the cell were needed to perform a heat balance on an element of the cylinder wall. Since the Sun is not a point source, however, the walls of a perfectly oriented honeycomb absorber will receive a contribution from the divergent solar rays. This contribution will increase as the honeycomb distance from the Sun decreases; that is, as the solar rays become more divergent. The following approach is used to estimate the effect on $\bar{\epsilon}_a$ of this additional heat input to the walls for a distance of 0.1 astronomical unit from the Sun:



Assume the Sun to be a disk with radius r, area A_2, and temperature T_s of 10 440° R (5800 K). The solar input to dA_1 (see sketch) is determined by using the following approach:

$$s^2 = h^2 + \rho^2 \tag{B1}$$

$$dA_2 = \rho \, d\rho \, d\alpha \tag{B2}$$

$$\cos \beta_1 = \frac{\rho \cos \alpha}{s} \tag{B3}$$

$$\cos \beta_2 = \frac{h}{s} \tag{B4}$$

The heat d^2Q radiated per unit time from surface dA_2 within the solid angle under which dA_1 is seen from dA_2 is

$$d^2Q = \frac{\sigma T_s^4}{\pi} \cos \beta_2 \, d\omega_2 \, dA_2 \tag{B5}$$

where T_s is the temperature of the Sun, 10 440° R (5800 K), and $d\omega_2$ is the solid angle under which dA_1 is seen from dA_2. The expression

$$d\omega_2 = \frac{dA_1 \cos \beta_1}{s^2} \tag{B6}$$

gives the solid angle. Thus,

$$d^2Q = \frac{\sigma T_s^4}{\pi} \frac{\cos \beta_1 \cos \beta_2}{s^2} dA_1 \, dA_2 \tag{B7}$$

(The heat radiated from dA_1 to dA_2 is negligible in this case.) Thus,

$$dQ = \frac{\sigma T_s^4}{\pi} \left(\int_{A_2} \frac{\cos \beta_1 \cos \beta_2}{s^2} dA_2 \right) dA_1 \tag{B8}$$

or

$$\frac{dQ}{dA_1} = \frac{\sigma T_s^4}{\pi} \int_{A_2} \frac{\cos \beta_1 \cos \beta_2}{s^2} \, dA_2$$

$$= \frac{\sigma T_s^4}{\pi} \int_{A_2} \frac{h \cdot \rho \cos \alpha}{s^2} \cdot \frac{\rho \, d\rho \, d\alpha}{s^2} = \frac{h\sigma T_s^4}{\pi} \int_{A_2} \frac{\rho^2 \, d\rho \cos \alpha \, d\alpha}{(h^2 + \rho^2)^2}$$

$$= \frac{2h\sigma T_s^4}{\pi} \int_0^r \frac{\rho^2 \, d\rho}{(h^2 + \rho^2)^2} \int_0^{\pi/2} \cos \alpha \, d\alpha$$

$$= \frac{2h\sigma T_s^4}{\pi} \int_0^r \frac{\rho^2 \, d\rho}{(h^2 + \rho^2)^2} \left(\sin \alpha \Big|_0^{\pi/2} \right)$$

$$= \frac{2h\sigma T_s^4}{\pi} \int_0^r \frac{\rho^2 \, d\rho}{(h^2 + \rho^2)^2}$$

$$= \frac{2h\sigma T_s^4}{\pi} \left\{ \frac{1}{2} \left[\frac{1}{h} \tan^{-1} \left(\frac{\rho}{h} \right) - \frac{\rho}{h^2 + \rho^2} \right] \Bigg|_0^r \right\}$$

$$= \frac{h\sigma T_s^4}{\pi} \left[\frac{1}{h} \tan^{-1} \left(\frac{r}{h} \right) - \frac{r}{h^2 + r^2} \right]$$

$$= \frac{\sigma T_s^4}{\pi} \left[\tan^{-1} \left(\frac{r}{h} \right) - \frac{hr}{h^2 + r^2} \right] \qquad \text{(B9)}$$

Evaluating equation (B9), where r is the radius of the Sun, 4.33×10^5 miles (6.96×10^{10} cm), and h is the distance from the Sun, 0.1 astronomical unit or 9.3×10^6 miles (1.496×10^{12} cm), yields

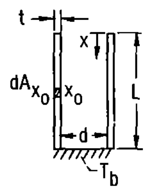
$$\frac{dQ}{dA_1} = 1.99\times10^{-5}\ \sigma T_s^4$$

$$\frac{dQ}{dA_1} = q_s = 2.8\ \mathrm{Btu/(hr)(in.^2)}\ (0.127\ \mathrm{W/cm^2})$$

APPENDIX C

CALCULATED EMITTANCE OF BLACK HONEYCOMB SOLAR ABSORBER AT 0.1 ASTRONOMICAL UNIT FROM SUN

The results of appendix B are used to determine the effect on $\bar{\epsilon}_a$ of an additional energy input to the walls of a honeycomb absorber at 0.1 astronomical unit from the Sun. The additional energy input is 2.8 Btu per hour - square inch (0.127 W/cm^2).



Consider a cavity with nonisothermal walls at 0.1 astronomical unit from the Sun. At equilibrium, an energy balance on a typical element of the cavity wall located at x_o and having an area $dA_{x_o} = (\pi d)dx_o$ is

$$\left(dQ_{net}\right)_{cond} + \left(dQ_{net}\right)_{rad} = 0 \tag{C1}$$

The temperature variation across the wall thickness t is assumed negligible

$$\left.\begin{aligned}\left(dQ_{net}\right)_{cond} &= -k(\pi d)t \frac{d^2T}{dx^2} dx_o \qquad \text{where} \quad k \neq k(T) \\ &= -kt \frac{d^2T}{dx^2} dA_{x_o}\end{aligned}\right\} \tag{C2}$$

The radiation emitted by the element is

$$\sigma T(x_o)^4 \, dA_{x_o} \tag{C3}$$

The incident radiation comes from three sources:

(1) Reradiation from the cylindrical surface of the body
(2) Reradiation from the base surface of the cavity
(3) Solar input due to divergent rays from the Sun

Reradiation from Cylindrical Surface of Body

The radiation leaving an element dA_x on the cylinder wall and arriving at dA_{x_o} is

$$\sigma T_{(x)}^4 \, dF_{x-x_o} \, dA_x \tag{C4}$$

where dF_{x-x_o} is the diffuse-angle factor between an element dA_x and an element dA_{x_o}. The reciprocity theorem for diffuse-angle factors states that

$$dF_{x-x_o} \, dA_x = dF_{x_o-x} \, dA_{x_o}$$

Therefore, expression (C4) becomes

$$\sigma T_{(x)}^4 \, dF_{x_o-x} \, dA_{x_o} \tag{C5}$$

The total contribution from the entire cylindrical wall to the incident energy at x_o is

$$dA_{x_o} \int_{x=0}^{L} \sigma T_{(x)}^4 \, dF_{x_o-x} \tag{C6}$$

Reradiation from Base Surface of Cavity

The radiation from the base surface to dA_{x_o} is analogously

$$\sigma T_b^4 F_{x_o-b} \, dA_{x_o}$$

or

$$\sigma T_b^4 F_{b-x_o} \frac{\pi d^2}{4} \tag{C7}$$

Solar Input Due to Divergent Rays From Sun

The solar input to the wall from the divergent rays of the Sun at 0.1 astronomical unit is estimated to be 2.8 Btu per hour - square inch (0.127 W/cm^2) = q_s. Thus, the total contribution due to the solar divergent radiation to dA_{x_o} is

$$q_s \, dA_{x_o} \tag{C8}$$

The net radiation from an element at x_o is then obtained by subtraction of expressions (6) to (8) from (3). Thus,

$$\left(dQ_{net}\right)_{rad} = \left[\sigma T^4_{(x_o)} - \int_{x=0}^{L} \sigma T^4_{(x)} \, dF_{x_o - x} - \sigma T_b^4 F_{x_o - b} - q_s\right] dA_{x_o} \tag{C9}$$

Substituting equations (2) and (9) into (1) yields

$$\left[\sigma T^4(x_o) - \int_{x=0}^{L} \sigma T^4_{(x)} \, dF_{x_o - x} - \sigma T_b^4 F_{x_o - b} - q_s\right] dA_{x_o} = kt \frac{d^2T}{dx^2} dA_{x_o} \tag{C10}$$

Substitute the following dimensionless variables:

$$X_o = \frac{x_o}{d}; \quad X = \frac{x}{d}; \quad \theta = \frac{T}{T_b}$$

And note that

$$\frac{d^2T(x)}{dx^2} = \frac{T_b}{d^2} \frac{d^2\theta(X)}{dX^2}$$

$$\sigma T_b^4 \theta^4(X_o) - \int_{X=0}^{L/d} \sigma T_b^4 \theta^4(X) dF_{X_o - X} - \sigma T_b^4 F_{X_o - b} - q_s = \frac{ktT_b}{d^2} \frac{d^2\theta(X)}{dX^2}$$

$$\sigma T_b^4 \left[\theta^4(X_o) - \int_{X=0}^{L/d} \theta^4(X) dF_{X_o - X} - F_{X_o - b} - \frac{q_s}{\sigma T_b^4}\right] = \frac{ktT_b}{d^2} \frac{d^2\theta(X)}{dX^2}$$

Thus,

$$\frac{d^2\theta(X)}{dX^2} = \frac{\sigma T_b^3 d^2}{kt}\left[\theta^4(X_o) - \int_{X=0}^{L/d} \theta^4(X)dF_{X_o-X} - F_{X_o-b} - \frac{q_s}{\sigma T_b^4}\right]$$

Equation (C11) is identical to equation (9), except for the added term $q_s/\sigma T_b^4$, and is solved in the same manner to give the temperature along the wall as a function of the distance from the opening x/d. The radiant flux q_t emitted from the cell can then be determined as in equation (15).

Performance calculations for the honeycomb absorber at 0.1 astronomical unit from the Sun are compared with collimated solar input conditions in figures 8 and 9 for the reference case, which has a base surface temperature T_b of 2060° R (1144 K), a cylinder diameter d of 0.5 inch (1.27 cm), a cell wall thickness t of 0.003 inch (0.00761 cm), and a thermal conductivity k of 0.667 Btu per hour-inch-°R (0.1384 W/(cm)(K)). As expected, the effect of the added energy input due to the divergent rays of the Sun at 0.1 astronomical unit is to increase the cell wall temperature and, hence, the value of $\overline{\epsilon}_a$ relative to the collimated solar input condition. In figure 8, for

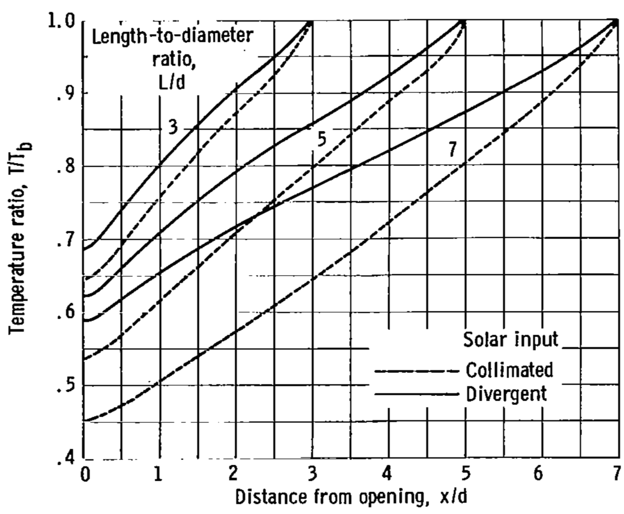


Figure 8. - Comparison of cell-wall temperature ratio for collimated and divergent solar input conditions for 0.1-astronomical unit distance from Sun. Reference conditions: base temperature, 2060° R (1144 K); diameter, 0.5 inch (1.27 cm); thermal conductivity, 0.667 Btu per hour-inch-°R (0.1384 W/(cm)(K)); cell wall thickness, 0.003 inch (0.00761 cm).

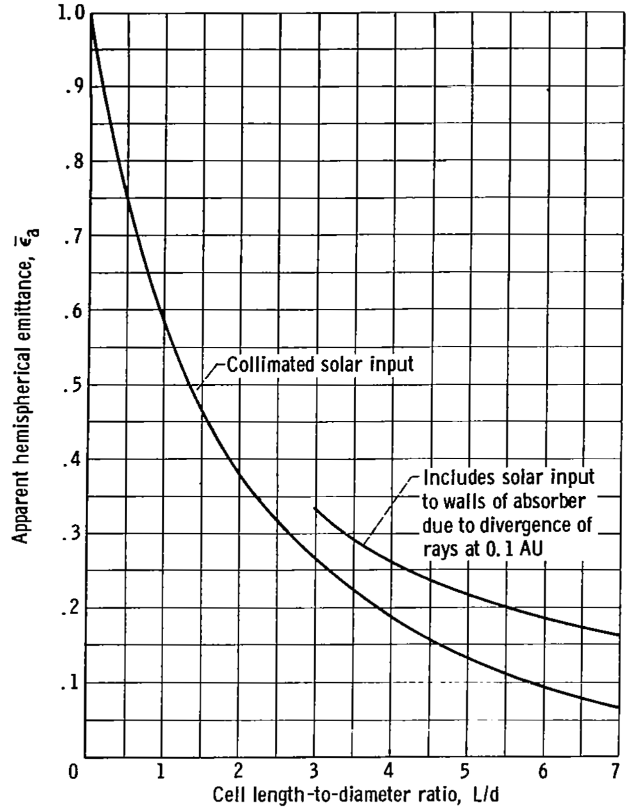


Figure 9. - Apparent hemispherical emittance as function of cell length-to-diameter ratio. Reference conditions: base temperature, 2060° R (1144 K); diameter, 0.5 inch (1.27 cm); thermal conductivity, 0.667 Btu per hour-inch-°R (0.1384 W/(cm)(K)); cell wall thickness, 0.003 inch (0.00761 cm).

example, at an L/d of 5, the temperature at the cell opening is 0.535 T_b for the collimated case, while for the 0.1-astronomical unit case, the temperature increases to 0.620 T_b. This increase in wall temperature near the cell opening results in a substantial increase in $\bar{\epsilon}_a$. For example, in figure 9, at an L/d of 5.75, $\bar{\epsilon}_a$ is 0.1 for the collimated case, whereas $\bar{\epsilon}_a$ is increased to 0.19 for the 0.1-astronomical unit case.

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